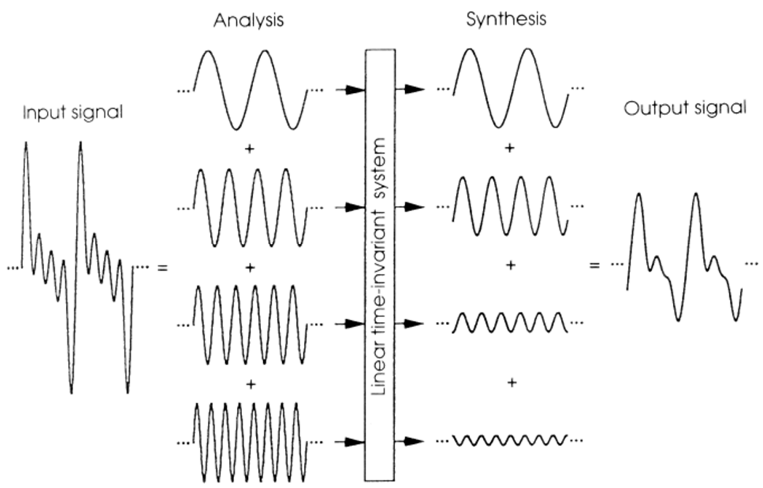
Signals & Systems for Speech & Hearing

Week 4

Representing signals as sums of sinusoids: Spectra

The big idea

As long as we know what the system does to sinusoids...



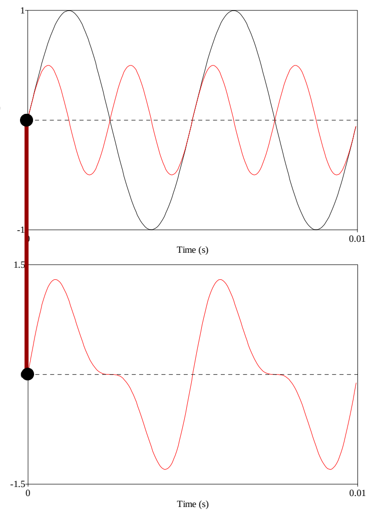
... we can predict any output to any input.

Synthesising waves

French mathematician
Jean Baptiste Joseph Fourier
1768-1830

Fourier Synthesis

we add up sinewaves by adding up the respective amplitude values of all sine waves for each point in time



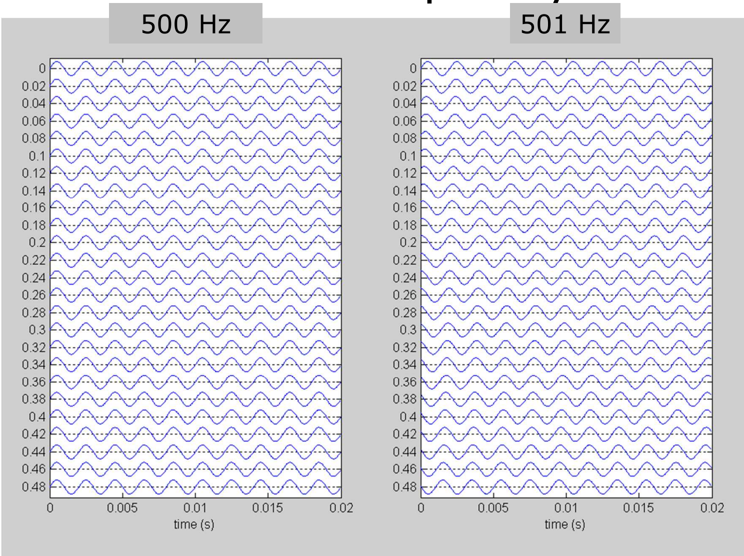
sinewave I: 200 Hz

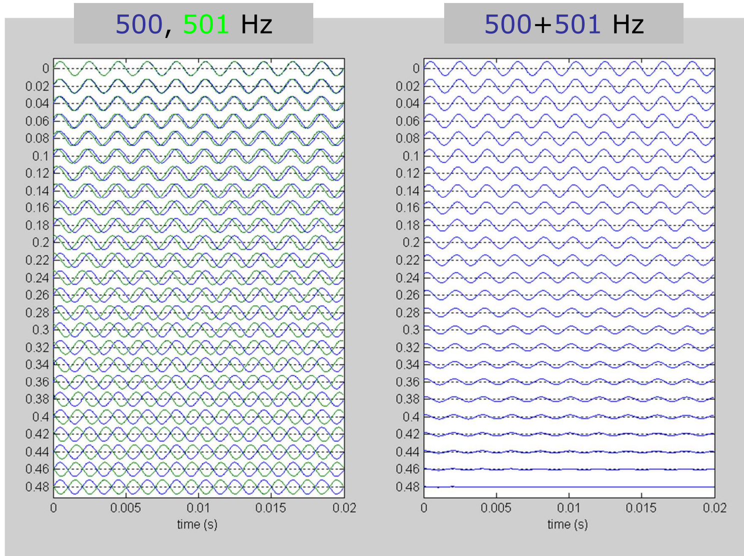
+

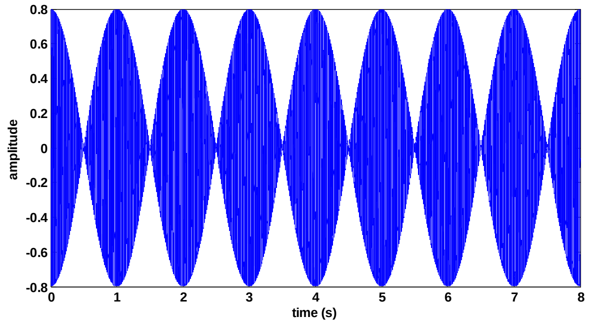
sinewave II: 400 Hz

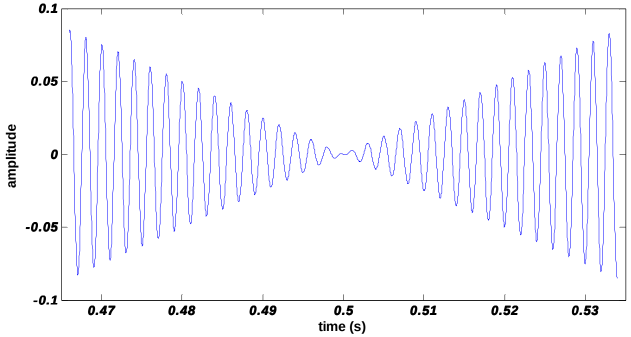
this leads to a complex waveform consisting of a 200 and a 400 Hz sinusoid

Beats: Add 2 sinewaves that are close in frequency









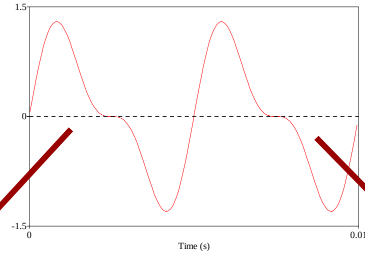
Fourier Analysis

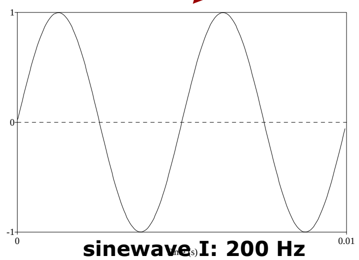
Fourier series analysis

(calculus based)

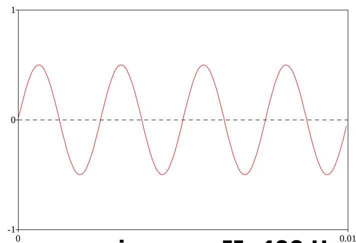
Suppose we are given a complex waveform:

The question is, which are the underlying sine waves?





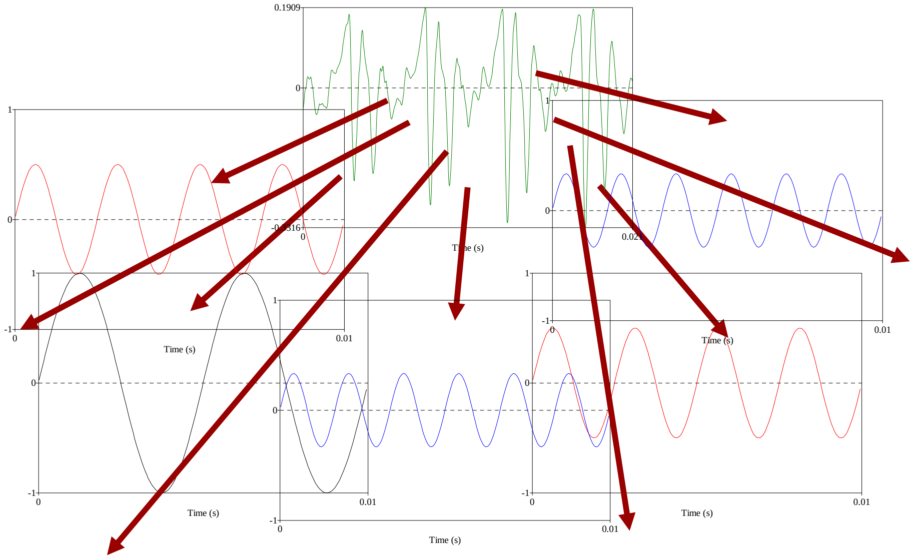
sinewave I: 200 Hz



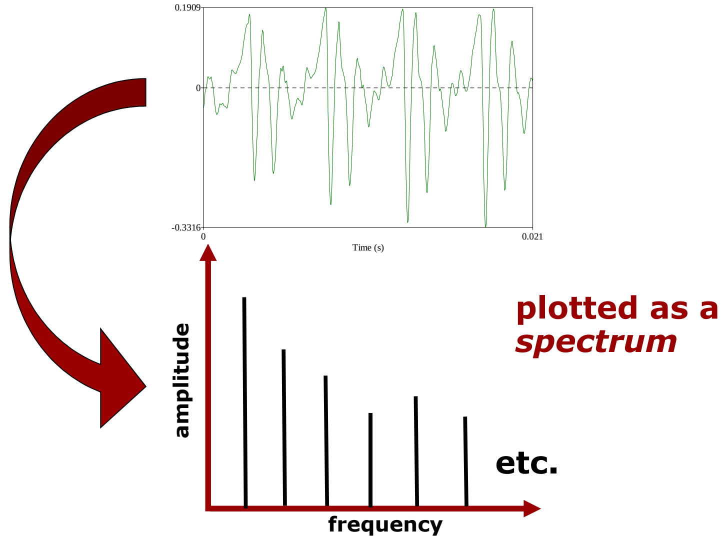
sinewave II: 400 Hz

Fourier Analysis

What if the complex wave is really complex?



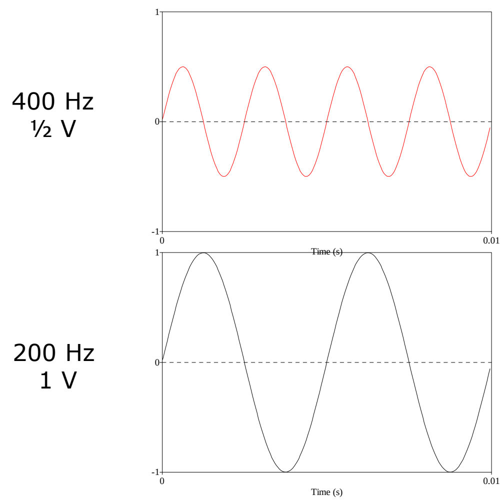
Fourier Analysis



How to determine a spectrum

- Easy to see how to *synthesise*
  - spectrum → waveform
- But how do we analyse?
  - waveform → spectrum
- A special case: periodic complex waves
  - All component sine waves must be ***harmonically*** related
  - Their frequencies must be integer (whole-number) multiples of the repetition frequency of the complex waveform

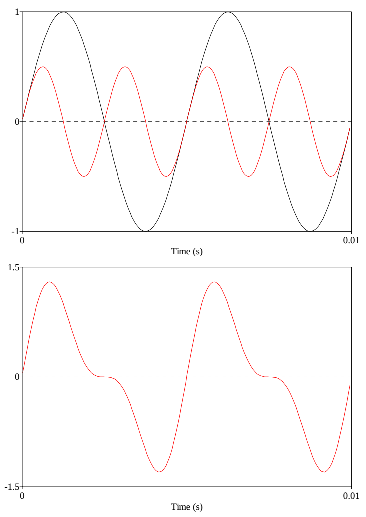
Adding more than two sinusoids: component sine waves



Adding Waveforms

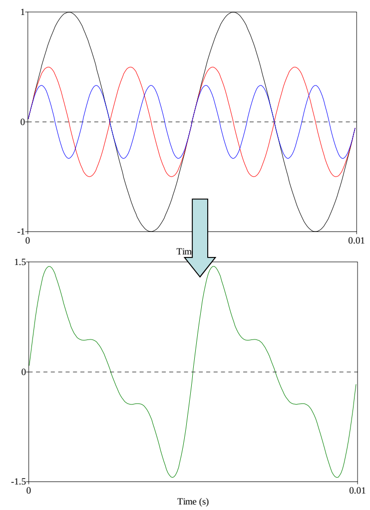
sinusoids

complex waveform

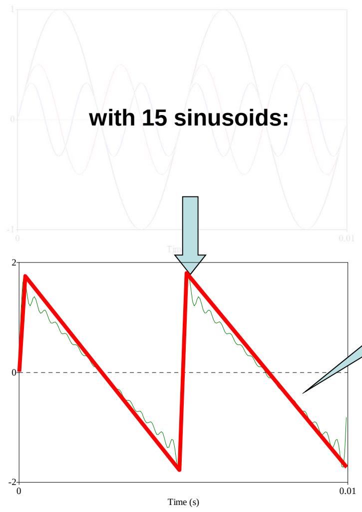


Adding a third sinusoid

resulting periodic complex wave

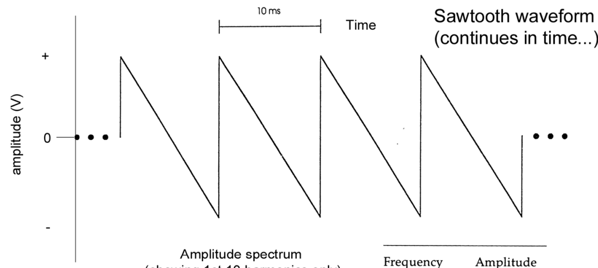


Adding 15 sinusoids

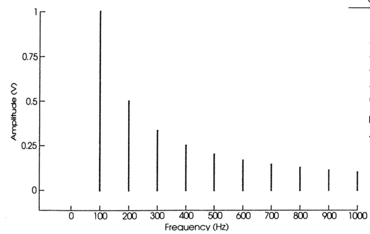


The resulting waveform becomes more and more like a **sawtooth**

Spectrum of the sawtooth waveform



Amplitude spectrum
(showing 1st 10 harmonics only)

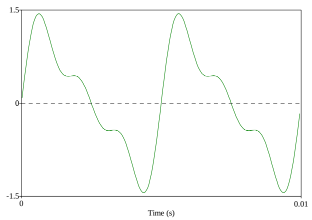


Frequency (Hz)	Amplitude (V)
100	1
200	1/2
300	1/3
400	1/4
500	1/5
600	1/6
n x 100	1/n
for n = 1 to infinity	

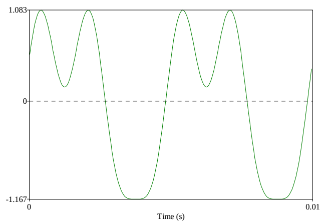
Visual effects of 'phase'

Phase can have a great effect on the resulting complex waveform, e.g.:

200, 400, and 600 Hz sinusoids added:



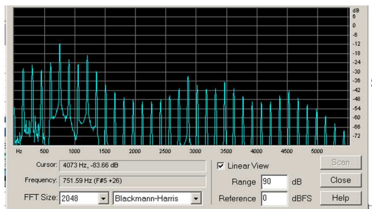
all in the same (sine) phase



400 Hz sinusoid is + 90°

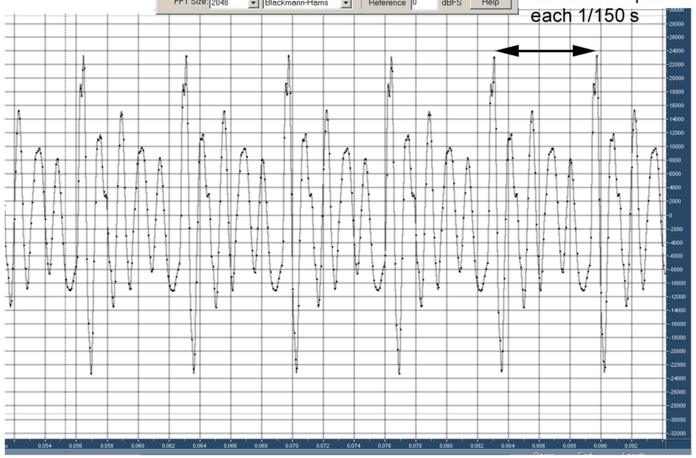
Other periodic complex waves

- Infinite number of possible periodic complex wave shapes.
- *All* complex periodic waves have spectra whose sine-wave components are *harmonically-related*
  - frequencies are whole-number (integer) multiples of a common "fundamental" frequency.



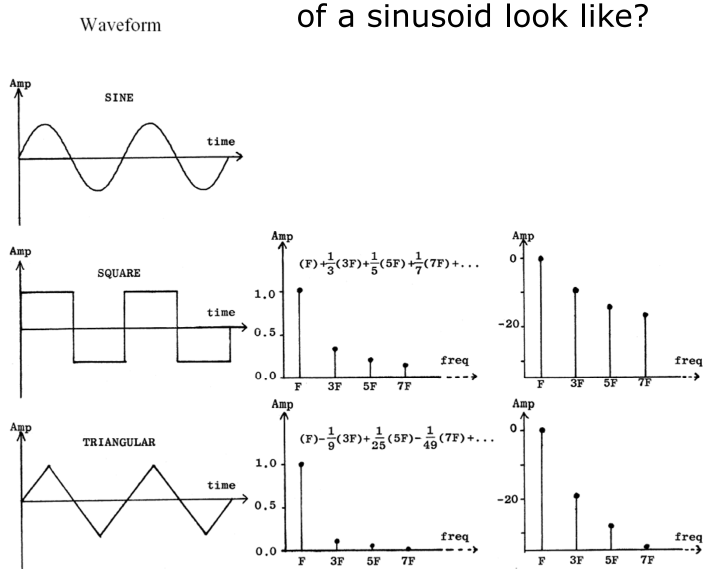
spectrum shows harmonics of 150 Hz

waveform repeats each 1/150 s



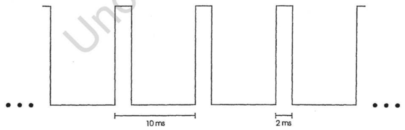
Vowel with fixed f_0

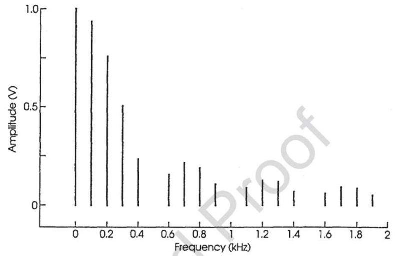
What does the spectrum of a sinusoid look like?



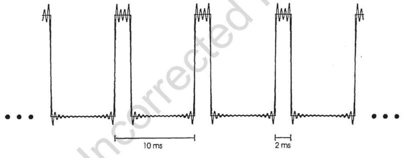
Spectrum of a pulse train

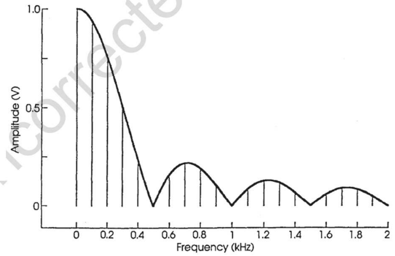
the original





the approximation





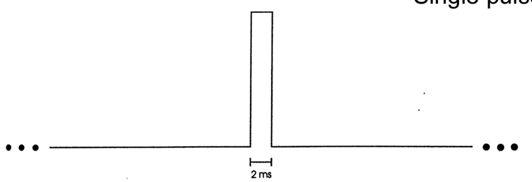
Spectra of periodic waves

- Only the possible frequencies are constrained. The amplitude and phase of each harmonic can have any possible value
  - including zero amplitude.
- Fundamental frequency (f0) is the *greatest common factor* of harmonic frequencies.
- Series of harmonics at:
  - 100, 200, 300 Hz: f0 = 100Hz
  - 150, 200, 250 Hz: f0 = 50Hz
  - 200, 700, 1000 Hz: f0 = 100Hz

Spectra of aperiodic waves

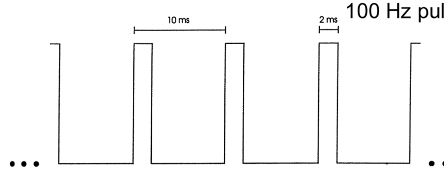
- Aperiodic waves can also be constructed from a series of sinusoids …
  - but not using harmonics only.
- Spectra are continuous – every possible frequency is present…
  - as if harmonics were infinitely close together.
- What is the spectrum of a single pulse?

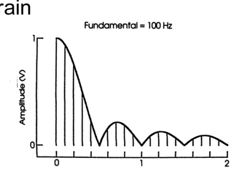
Single pulse



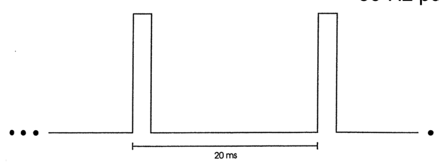
Keep lowering the fundamental frequency of a train of pulses

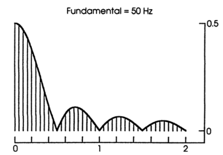
100 Hz pulse train



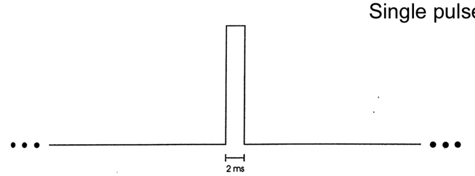


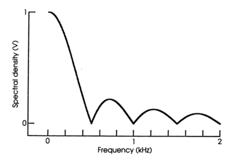
50 Hz pulse train



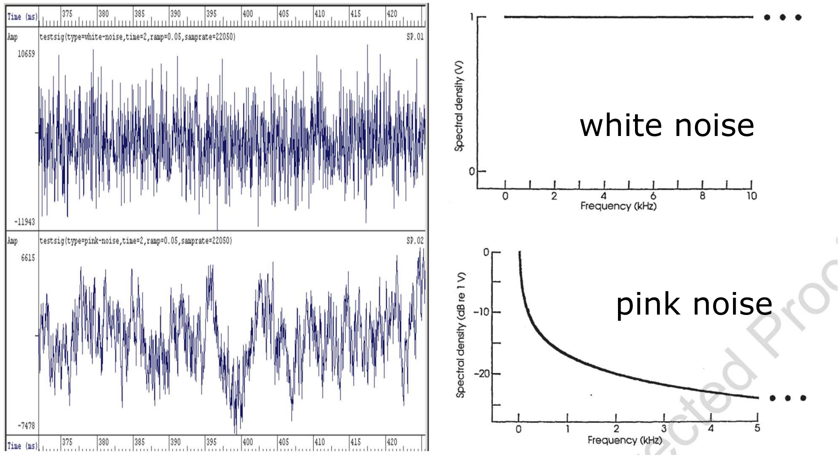


Single pulse





Spectra of random aperiodic sounds



Q: Why 'white' and 'pink'?

Q: Why 'white' and 'pink'?
A: analogies to light waves

frequency (Hz)

4×10^{14} — 7.5×10^{14}

400 THz — 750 THz

kilo-	k	10^3
mega-	M	10^6
giga-	G	10^9
tera-	T	10^{12}
peta-	P	10^{15}

Key Points

- Fourier synthesis
  - any waveform can be constructed by adding together a unique series of sine-waves, each specified by frequency, amplitude and phase …
  - but an infinite number may be needed.
- Fourier analysis
  - Any waveform can be decomposed into a unique set of component sinusoids
  - involves complex mathematics but this is easily carried out by computers and digital signal processors.
- Periodic waves have spectra that can only consist of components at harmonic frequencies of the fundamental.
- Aperiodic waves can have anything else – almost always *continuous* spectra.